

## ACT: Mathematics Formulas

**Math Concepts to Review:** decimals/place value, fractions/mixed numbers, integers (...-3, -2, -1, 0, 1, 2, 3...), scientific notation, factors (divide into a number without a remainder), multiples (skip count by a given number), ratios, proportions, percents (literally means "per 100"), P(G)EMDAS (order of operations), types of numbers (rational, irrational...), prime/composite, absolute value, square roots

**Arithmetic Sequences:**  $t_n = t_1 + d(n - 1)$  {e.g. 1, 3, 5, 7... where (d) is the common difference}

**Geometric Sequences:**  $t_n = t_1 r^{(n-1)}$  {e.g. 1, 5, 25, 125... where (r) is the common ratio}

**Statistics:** mean (average;  $\frac{\text{sum of values}}{\text{number of values}}$ ), median (middle number when data is arranged from least to greatest; find average of middle two if no single center data point), mode (value in data which occurs most in set), range (difference between max and min value in data set)

**Probability:**  $\frac{\text{favorable outcomes}}{\text{possible outcomes}}$

**Combinations:** multiply choices

**Average Speed:**  $d=rt$

**Work:**  $w=rt$       **Combined Work:**  $w = (r_1 + r_2)t$

**Exponents & Roots:**

$x^0 = 1$	$\frac{x^a}{x^b} = x^{a-b}$	$(x^a)^b = x^{a \cdot b}$	$(xy)^a = x^a \cdot y^a$
$x^a \cdot x^b = x^{a+b}$	$x^{-a} = \frac{1}{x^a}$	$\sqrt[a]{x} = x^{\frac{1}{a}}$	$x^{\frac{b}{a}} = \sqrt[a]{x^b}$

**Lines:** SLOPE OF A LINE  $m = \frac{y_2 - y_1}{x_2 - x_1}$     or     $\frac{\Delta y}{\Delta x}$     or     $\frac{\text{rise}}{\text{run}}$

PARALLEL lines = same slope

PERPENDICULAR lines = slopes are negative reciprocals

SLOPE-INTERCEPT FORM  $y = mx + b$

POINT-SLOPE FORM  $y - y_1 = m(x - x_1)$

MIDPOINT FORMULA  $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$

DISTANCE FORMULA  $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

### Exponential Functions:

Growth/Decay {change percent (r) to decimal}

$$A(t) = A_o(1 \pm r)^t$$

Interest {n times for t years,  $A_o$  is initial value}

$$A(t) = A_o \left(1 \pm \frac{r}{n}\right)^{nt}$$

**Domain:** set of possible x values

**Range:** set of possible y values

**System of Equations:** use elimination (linear combination/subtracting) or substitution; if answer is true there are infinite solutions (lines are same & slope and y-intercept are same), if answer is false there are no solutions (lines are parallel & slope is same but y-intercept is different),  $x = a$  # if lines intersect at one point

**Factorial:** 5! is read "5 factorial" and is solved by  $5 \times 4 \times 3 \times 2 \times 1$ ...most calculators have a factorial function

**Complex Numbers:** in the form  $a + bi$  where  $i = \sqrt{-1}$ ,  $i^2 = -1$ ,  $i^3 = -i$ ,  $i^4 = 1$

Multiply by the conjugate to rationalize the denominator:  $(3 + 4i)(3 - 4i) = (9 - 16i^2) = 25$

**Logarithms:**  $y = \log_b x$  converts to exponent form  $b^y = x$

$$\log_b b^n = n \qquad \log_b xy = \log_b x + \log_b y$$

$$\log_b x^y = y \log_b x \qquad \log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$$

Change of base formula

$$\log_b(a) = \frac{\log_x(a)}{\log_x(b)}$$

**Parabolas:** STANDARD FORM  $y = ax^2 + bx + c$  Factor with "reverse FOIL" to solve  $x^2 + (b+a)x + ab = (x+a)(x+b)$

or use the QUADRATIC FORMULA  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  vertex =  $(-\frac{b}{2a}, \text{plug value back into equation})$

VERTEX FORM  $y = a(x - h)^2 + k$  vertex =  $(h, k)$

FACTORED/  
INTERCEPT FORM  $y = a(x - p)(x - q)$  x-intercepts/zeros/solutions =  $(p, 0)$  &  $(q, 0)$   
vertex = (midpoint of two zeros, plug value back in to find y)

DISCRIMINANT This is the expression under the radical in the Quadratic Formula

if  $b^2 - 4ac > 0$  then 2 real solutions

if  $b^2 - 4ac = 0$  then 1 real solution

if  $b^2 - 4ac < 0$  then 0 real solutions, 2 imaginary

**Circles:** AREA OF CIRCLE  $A = \pi r^2$  AREA OF SECTOR  $\frac{n^\circ}{360} \cdot \pi r^2$

CIRCUMFERENCE OF CIRCLE  $C = 2\pi r$  LENGTH OF ARC  $\frac{n^\circ}{360} \cdot 2\pi r$

Circle =  $360^\circ = 2\pi$  radians Degrees to Radians =  $\text{degrees} \cdot \frac{\pi}{180}$  Radians to Degrees =  $\text{radians} \cdot \frac{180}{\pi}$

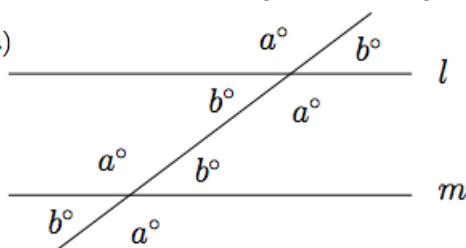
STANDARD FORM EQUATION OF A CIRCLE with center  $(h, k)$  and radius  $r$  is  $(x - h)^2 + (y - k)^2 = r^2$

CENTRAL ANGLES are congruent to corresponding arc; INSCRIBED ANGLES are half of their corresponding arc measure.

**Angles:** SUM OF INTERIOR ANGLES  $180(n - 2)^\circ$  SUM OF EXTERIOR ANGLES  $360^\circ$

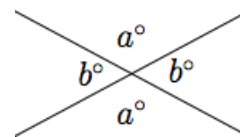
INTERSECTING LINES vertical (opposite) angles are congruent and linear pairs add to  $180^\circ$

PARALLEL LINES ( $l \parallel m$ )  
alternate interior,  
alternate exterior,  
and corresponding  
angles are  
congruent

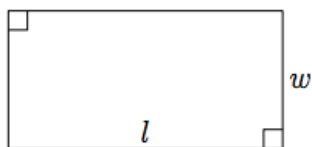


COMPLEMENTARY-angles  
sum to  $90^\circ$

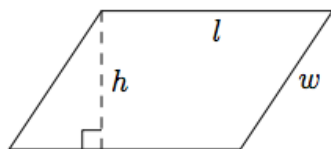
SUPPLEMENTARY-angles  
sum to  $180^\circ$



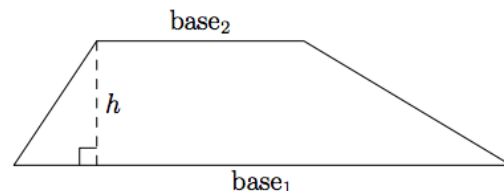
**Perimeter/Area/Volume:** To find PERIMETER of any shape, simply add all side lengths.



Rectangle  
(Square if  $l = w$ )  
Area =  $lw$



Parallelogram  
(Rhombus if  $l = w$ )  
Area =  $lh$



Area of trapezoid =  $\left(\frac{\text{base}_1 + \text{base}_2}{2}\right) \cdot h$

VOLUME OF ANY PRISM/CYLINDER = Area of Base x Height VOLUME OF PYRAMID/CONE =  $\frac{1}{3}$  Area of Base x Height

SURFACE AREA OF ANY PRISM/PYRAMID = Area of Bases + Area of Sides

SURFACE AREA OF CYLINDER =  $2(\pi r^2) + 2\pi r(h)$

SURFACE AREA OF CONE =  $\pi r^2 + \pi rl$  where  $r$  = radius and  $l$  = slant height

VOLUME OF SPHERE =  $V = \frac{4}{3}\pi r^3$  SURFACE AREA OF SPHERE =  $4\pi r^2$

DIAGONAL OF ANY PRISM =  $d = \sqrt{l^2 + w^2 + h^2}$

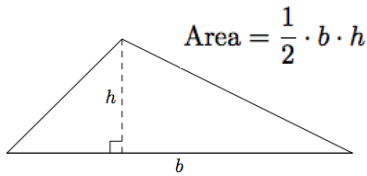
**Triangles:** TRIANGLE SUM THEOREM-The sum of the three angles in any triangle must equal  $180^\circ$ .

EXTERIOR ANGLE THEOREM- The exterior angle of a triangle is equal to the sum of the remote interior angles.

TRIANGLE INEQUALITY THEOREM- The sum of any two sides of a triangle must be greater than the third side.

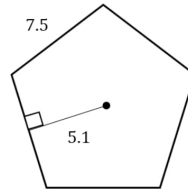
TRIANGLE SIMILARITY- corresponding angles are congruent and sides are in proportion (AA, SSS, SAS)

TRIANGLE CONGRUENCY- all corresponding angles and sides are congruent (SSS, ASA, SAS, HL)



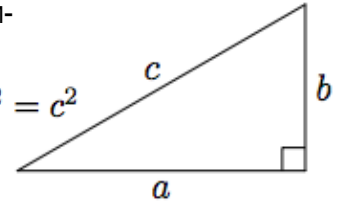
AREA OF REGULAR POLYGONS-

find the area for one triangle and multiply by the number of triangles in polygon



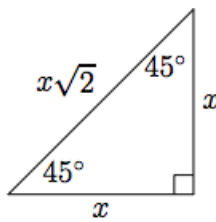
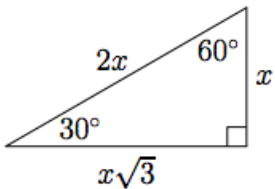
PYTHAGOREAN THEOREM-

$$a^2 + b^2 = c^2$$



Pythagorean Triples- consist of three integers which satisfy the PYTHAGOREAN THEOREM (3,4,5) (5,12,13) (8,15,17) (7,24,25)

SPECIAL RIGHT TRIANGLES-



**Trigonometry: "SOH-CAH-TOA"**

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin \theta}{\cos \theta}$$

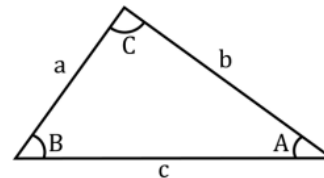
$$\frac{1}{\sin \theta} = \csc \theta$$

$$\frac{1}{\cos \theta} = \sec \theta$$

$$\frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta} = \cot \theta$$

TRIGONOMETRY IDENTITY-  $\sin^2 \theta + \cos^2 \theta = 1$

LAW OF SINES-  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



LAW OF COSINES-  $c^2 = a^2 + b^2 - 2ab \cos(C)$

